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Intuition in Poincaré's Philosophy of Mathematics

Poincaré'nin Matematik Felsefesinde Sezgi

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Abstract: This paper aims to shed light on Henri Poincaré's intuitionism. Today, the intuitionistic philosophy of mathematics is usually associated with L.E.J. Brouwer and the idea of expelling the proofs which rest on the law of excluded middle from mathematics. There is a widespread supposition that intuitionists argue that there is a certain error in our standard way of doing mathematics and that a radical change in mathematics is needed. It is interesting to note, however, that Immanuel Kant, who was the first philosopher to relate mathematics to the intuitions of the human being, did not maintain such an argument and he used the term intuition in a different sense than what is generally understood today. This is also true of Poincaré, who made a significant revision to Kant's philosophy of mathematics and who is usually regarded as a pre-intuitionist or a semi-intuitionist. Some philosophers, such as Warren Goldfarb, rightly argued that Poincaré's concern in invoking intuition was to explain the psychological aspect of mathematical thinking. It is argued in this paper that this psychological aspect was not the whole point of Poincaré's intuitionism as there is a notion of a pure, a priori intuition in his philosophy which he borrowed from the Kantian tradition.

Keywords: Intuitionism, Kant, Poincaré, synthetic a priori, mathematical induction.

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1. Philosophy of Arithmetic and Geometry: Kant and After Kant

Just as for many other branches of philosophy, Immanuel Kant's philosophy marked a new beginning for the philosophy of mathematics. Kant argued that the truth of mathematical propositions does not rest upon the sensible objects that we encounter in experience, but rather upon the forms that lie ready in the mind prior to all experience and to which every object of experience must conform. These forms are *space* and *time* – in Poincaré's words, they are mental frameworks (2011, p. xix). According to Kant, space and time are the pure forms of our faculty of sensibility and we have a direct intuition of these forms. In Kant's philosophy, the necessity and universality of mathematical propositions are explained on the basis of the existence of such pure forms, which are given prior to all experience and which furthermore stand as the conditions of the possibility of experience. For Kant, what guides us in deciding whether a proposition of arithmetic is true or false is neither logic nor the sensible objects that we encounter in experience, but the intuition of time, because, ultimately, a natural number is understood as a successive addition of units in time. Similarly, what guides us in geometry is the intuition of space, because a geometric figure is never given in experience but rather constructed in pure intuition.

Kant's critical method and his ideas concerning almost every branch of philosophy have attracted much attention from subsequent philosophers. Naturally, his 'intuitionistic' philosophy of mathematics received its own share of this attention. However, in the course of time the conviction in the Kantian idea that there exist in our minds frameworks which are independent of experience was weakened in the light of the invention of non-Euclidean geometries and the theory of evolution by natural selection.

Among the first people to take the possibility of non-Euclidean geometries seriously and to work on them was Carl Friedrich Gauss. In 1792, when he was only 15 years old, Gauss started to work on the fifth postulate of Euclid[1], and in 1817 he was convinced that this postulate was

[1] "If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines will intersect each other on that side if extended far enough". This postulate is equivalent of what is known as

independent of the other four postulates, i.e., that it could not be derived from them. He then started to work on alternative geometries where this postulate was rejected, yet he never published those works. The first publication concerning non-Euclidean geometries came from the Hungarian mathematician Janos Bolyai in 1825, in the form of an appendix to his father Farkas Bolyai's book, who was a close friend of Gauss. Russian mathematician Nikolai Lobachevsky, unaware of Bolyai's work, published his own studies in Russian in 1829 in a local university publication, *Kazan Messenger*. The works of these two mathematicians were discussed in a rather small circle, but this changed in 1854 when Bernhard Riemann gave an inaugural lecture about his works on non-Euclidean geometries in which he completely reformulated the notion of space. Finally in 1868, Eugenio Beltrami set these non-Euclidean geometries on a firm basis and reduced the problem of their consistency to the problem of the consistency of Euclidean geometry.

These mathematicians showed that Euclid's postulates were not the only candidates for constituting a consistent system of geometry, and that it was possible to have different geometries each describing a particular space, even though these spaces may seem unintuitive. In these geometries the parallel postulate is replaced by other postulates[2], and so the results derived from the parallel postulate are false. For instance, in non-Euclidean geometries, the sum of interior angles of a triangle adds up to something greater or less than two right angles. Furthermore, two triangles having the same interior angles but different side lengths cannot be drawn – in other words, there are no similar triangles in these geometries – yet they are as consistent and as rich as Euclidean geometry. Schiller (1896) wrote, "If it is a universal and necessary truth that the angles of a triangle are equal to two right angles, it cannot be an equally universal and necessary truth that they are greater" (p. 179). Philosophers were thus

the *parallel postulate*, which simply states that given a line and a separate point, there exists only one line that passes through this point and never intersects the given line.

2 These are Lobachevsky's and Riemann's postulates. Lobachevsky's postulate states that there exist two lines parallel to a given line through a given point not on the line. Riemann's postulate states that there are no parallel lines. Riemann also had to reject the second postulate of Euclid, which states that a line segment can be extended indefinitely. Riemann assumed that it cannot, and on these suppositions he laid the foundations of spherical geometry.

led to question the existence of an *a priori* framework that our minds imposed upon experience inexorably, giving rise to Euclid's postulates, for non-Euclidean geometries clearly showed that there were other logically possible frameworks for describing space. Naturally, the following questions were raised: What is it that has led us into treating Euclidean geometry as intuitive; that is, why *this* form of sensibility rather than another? And among these geometries, which one is the *true* geometry?

The second reason why the conviction in the Kantian idea of a pure form of sensibility was weakened is the theory of evolution of biological species by natural selection, which had been becoming increasingly dominant[3]. The path that led to this theory was opened chiefly by the French biologist Jean-Baptiste Lamarck and his theory of inheritance of acquired characteristics, which first appeared in 1801. Almost sixty years later, the theory of evolution by natural selection was first formulated by Charles Darwin in his book *On the Origin of Species* (1859). With the help of the technological advancements of 19th and 20th centuries, but more importantly, with the establishment of modern biology and genetics, the number of observations confirming the idea that different traits in biological species are the result of mutation, adaptation, and selection, continued to spread. Philosophers then naturally questioned whether the frameworks we impose upon experience or the 'pure' intuitions we possess were shaped by evolution, as in the case of physical and behavioral traits, meaning that they were not that pure after all[4].

[3] In fact, in *Critique of the Power of Judgment*, Kant argues for something like a proto-Darwinian theory of evolution, although not yet for natural selection. He suggests that merely mechanical means can account for the variation in biological species: "The agreement of so many genera of animals in a certain common schema [...] strengthens the suspicion of a real kinship among them in their generation from a common proto-mother, through the gradual approach of one animal genus to the other, from that in which the principle of ends seems best confirmed, namely human beings, down to polyps, and from this even further to mosses and lichens, and finally to the lowest level of nature that we can observe, that of raw matter: from which, and from its forces governed by mechanical laws [...] the entire technique of nature [...] seems to derive" (2000, p.287, 5:419). But apparently Kant did not consider the possibility that the form of our sensibility or understanding could have such an origin; the hypothesis can be used to explain only the physical constitution of living beings, and even this was regarded by Kant as a "daring adventure of reason".

[4] A similar view was expressed by Poincaré in *Science and Method*, where he mentioned the role of adaptation and natural selection in acquiring the idea of space. According to him, the distinctive movements which allow us to parry incoming threats or reach desired objects are constitutive of space: "Certain hunters learn to shoot fish under the water, alt-

Seeing the problems with founding mathematics on our forms of sensibility and the pure intuitions of these forms which are supposedly independent of experience, philosophers sought another basis on which they could ground mathematics. This led some philosophers like Gottlob Frege to reject Kant's position and attempt to reduce mathematical principles to principles of logic. Others, such as David Hilbert, held that mathematics was simply the study of formal systems whose principles are like the rules of a game which are otherwise meaningless. Still others were committed to intuitionism but sought to revise Kant's original position. Henri Poincaré was a member of the last group; he remained a Kantian and an intuitionist. Though Poincaré made significant changes in Kant's thought – concerning mostly the origin of geometry and form of space – he favored the Kantian concepts of synthetic *a priori* and *pure intuition*, which were usually rejected by the other schools. He accepted that there are propositions in mathematics that are synthetic and known *a priori* on the basis of a pure intuition, but as Janet Folina (1986) writes, compared to Kant, "Poincaré's theory of the synthetic *a priori* is much more minimal" (p. 30). Some philosophers, such as Warren Goldfarb, see Poincaré's intuitionism as a psychological account of mathematical thinking, and seem to ignore the Kantian side of his intuitionism. Although Poincaré wrote on the psychology of the mathematician, he nevertheless mentioned a pure intuition which is prior to all experience and which gives rise to mathematical reasoning.

2. Poincaré against Logicism and Formalism

Poincaré presented his views in three books: *Science and Hypothesis* (1903), *The Value of Science* (1905), and *Science and Method* (1908). In these books he formulated his idiosyncratic intuitionism and mostly defended it against Bertrand Russell (1872-1970) and David Hilbert, the champions of logicism and formalism of his era. A while later than Kant, there emerged the logicist attempts, mainly by Frege and Russell, to free mathematics from any need for intuition and to reduce it to logic, thereby showing

hough the image of these fish is raised by refraction; and, moreover, they do it instinctively. Accordingly they have learnt to modify their ancient instinct of direction or, if you will, to substitute for the association A1, B1, another association A1, B2, because experience has shown them that the former does not succeed." (2008, p. 116).

that mathematical propositions are analytic *a priori*. Logicist philosophers thought that something could be known *a priori* only by virtue of its lack of factual content; there were no *a priori* intuitions that could serve as a basis for synthetic judgments. In truth, mathematical reasoning was not different than logical reasoning and it had nothing to do with forms of sensibility or pure intuitions. A. J. Ayer, for instance, who was a logicist, wrote, "To say that a proposition is true *a priori* is to say that it is a tautology" (1964, p. 301) and mathematics is only a "special class of analytic propositions, containing special terms" (p. 297).

Poincaré was one of the most formidable opponents of this tradition. He claimed that mathematics is not a gigantic tautology but rather a *science,* and contrary to logic, it has a "creative virtue" (2011, p. 3). In *The Value of Science*, Poincaré likens a logicist – whose only tool is *analysis*[5] – to a person who checks whether each move is made in accordance with the rules of the game in order to understand a game of chess. He argues that the person must rather recognize the strategy and the plan behind every move in order to truly understand the game. "We need a faculty which makes us see the end from afar, and intuition is this faculty" (p. 22). There is a parallel between chess and mathematics, in the sense that both are rule following procedures yet analysis alone is not sufficient for understanding either of them. Just like ascertaining the correctness of every move in a game of chess is not sufficient, neither is ascertaining every step of a mathematical proof:

> When we have examined these operations one after the other and ascertained that each is correct, are we to think we have grasped the real meaning of the demonstration? Shall we have understood it even when, by an effort of memory, we have become able to repeat this proof by reproducing all these elementary operations in just the order in which the inventor had arranged them? Evidently not; we shall not yet possess the entire reality; that I know not what which makes the unity of the demonstration will completely elude us (1907, p. 22).

The quote certainly ends in an obscure manner regarding what intuition is, but Poincaré clarified his views in the subsequent chapters, and so will we save the clarification for the last. Here, it is necessary to add that

[5] Referred to as "*division and dissection*" (1907, p. 23).

there is obviously a limit to the analogy between chess and mathematics: the former can "never become a science, for the different moves of the same piece are limited and do not resemble each other" (2011, p. 21).

Poincaré raised a similar criticism against Hilbert's formalist program. Hilbert wished to reduce the number of the fundamental assumptions of geometry to a minimum. Some of these assumptions might be understood intuitively, but Hilbert held that in essence they were simply formal rules from which theorems could be deduced by purely analytic procedures. There were others, such as Giuseppe Peano, who have tried to accomplish what Hilbert did in geometry for arithmetic and analysis. At the very beginning of his *Foundations of Geometry*, Hilbert (1950) wrote:

> Let us consider three distinct systems of things. The things composing the first system we will call *points* [...] those of the second we will call *straight lines* [...] and those of the third system we will call *planes* [...] We think of these points, straight lines, and planes as having certain mutual relations, which we indicate by means of such words as "are situated", "between", "parallel", "congruent", "continuous", etc. The complete and exact description of these relations follows as a consequence of the *axioms of geometry*.

Even though Poincaré admitted that he thought very highly of Hilbert's book, he still condemned Hilbert's approach. Regarding the 'things' Hilbert considered at the beginning of his book, Poincaré writes: "What these 'things' are we do not know, and we do not need to know – it would even be unfortunate that we should seek to know; all that we have the right to know about them is that we should learn their axioms" (2008, p. 122). And again:

> [In Hilbert's Formalism] in order to demonstrate a theorem, it is neither necessary nor even advantageous to know what it means. The geometer might be replaced by the *logic piano* imagined by Stanley Jevons; or, if you choose, a machine might be imagined where the assumptions were put in at one end, while the theorems came out at the other, like the legendary Chicago machine where the pigs go in alive and come out transformed into hams and sausages. No more than these machines need the mathematician know what he does (2008, p. 147).

From the lines above, we can see that Poincaré's emphasis was principally on *understanding*. If we completely neglect our intuitions that play a role in mathematics and adopt a strong formalist standpoint, then we would sacrifice an integral part of mathematics: we would "not divine by what caprice all these [theorems] were erected in this fashion one upon another" (1907, p. 22), and we would not see why among countless possible assumptions these particular ones were judged preferable to others (2008, p. 148). It would therefore be very difficult, if not impossible, to learn and understand mathematics if it is presented to us as a purely formal practice, and this is why Poincaré wrote that he would not recommend Hilbert's book to a schoolboy (2008, p. 122).

There is another perhaps even more serious criticism that Poincaré raised against Hilbert's program, and consequently against any other program that aims to prove the consistency of mathematics within a formal system, such as Peano's or Zermelo-Fraenkel's axiomatic systems. Poincaré argued that for all branches of mathematics, the principle of mathematical induction is an indispensable tool, which states that if a theorem is true for $\gamma = 1$ and if it is shown to be true for $\gamma + 1$ when it is true for an arbitrary γ, then the theorem is true for all natural numbers. Now where does this principle come from? If it is a purely logical principle, then its negation must be capable of being reduced to the principle of contradiction. But how can we be sure that this principle never leads to a contradiction? Poincaré maintained that every attempt to show that mathematics is consistent needs to prove, at some point, that the principle of mathematical induction is exempt from contradiction. Since this principle states something about an infinite number of cases, a direct verification showing that the principle is true for a finite number of cases would not suffice. "We must then have recourse to processes of demonstration, in which we shall generally be forced to invoke that very principle of complete induction that we are attempting to verify" (2008, p. 153). Thus, every attempt to prove the consistency of the principle of induction will make use of the principle itself, "for that is the only instrument which enables us to pass from the finite to the infinite" (2011, p. 14). This poses a problem for formalists and logicists, yet it is not a problem for an intuitionist like Poincaré, because just like Kant, Poincaré thought that this principle is an *a priori* synthetic judgment and is grounded upon a pure intuition; we immediately become conscious of its validity because it pertains to the constitution of our mind and we have a direct intuition of this.

After examining the quotations above, it might seem as if Poincaré used the term intuition in several different ways. In fact, he admitted that the meaning of this term was quite vague and he tried to elucidate it. In *The Value of Science* he writes, "To make any science, something else than pure logic is necessary. To designate this something else we have no word other than *intuition*. But how many different ideas are hidden under this same word?" (1907, p. 19). We saw that he used the term intuition to designate the faculty *that which makes us see the end from afar*. For Poincaré, this faculty is an integral part of understanding. A person who has not developed this kind of intuition in a particular field will lack something very crucial in comparison to a person who has, even though both are bound by the same rules and doing the same operations. Seen under this light, intuition appears to be something psychological. This led philosophers like Goldfarb (1988) to claim that Poincaré's concern in invoking intuition in mathematics was to explain the psychology of mathematical thinking. However, though an accurate observation, this is only one-half of Poincaré's intuitionism. Intuition understood this way is not exclusive to the mathematician; a chess player, a composer, and even a logicist requires its aid. What is intuited in all these practices is a certain strategy peculiar to that field. It is developed through many experiences and it allows the practitioner to immediately see beforehand what steps she should take. But Poincaré mentions another kind of intuition, one that is pure and reminiscent of Kant's version, which gives rise to mathematical reasoning. What is intuited here is not the strategy or the plan in this or that practice, and unlike Kant, Poincaré did not relate this to the intuition of the pure forms of our sensibility; this is rather the intuition of a certain power of the mind, which consists in "conceiving the indefinite repetition of the same act, when the act is once possible. The mind has a direct intuition of this power, and experiment can only be for it an opportunity of using it" (2011, p. 16). This intuition is pure, and unlike the intuition a chess player has, it is given prior to all experience. This is actually the intuition mentioned in the previous paragraph, i.e. the one to which we owe the principle of mathematical induction. Let us now specify the two kinds of intuition as Poincaré formulated them, and distinguish the one that serves as a basis to synthetic judgments in arithmetic, giving rise to the *science of number*.

3. Poincaré's Intuitionism

It is true that one of Poincaré's intentions was to explain the psychology of the mathematician and that he used the term *intuition* in order to do this. There are many passages where he described what goes on in the "soul" of a mathematician. The most informative passages of this kind are found in *Science and Method* (2008, pp.53-65). In the third chapter of the first book, Poincaré explains the intuition that guides the mathematician in mathematical invention by describing in detail the mental processes after which he managed to establish the existence of different classes of Fuchsian functions. Reflecting on his experiences, Poincaré concludes that both conscious and unconscious mental procedures play a role in mathematical invention. The conscious procedures consist of many trials with different combinations, carried out gropingly. These are indispensable for a mathematical proof, since through them the mathematician gains an acquaintance with his problem and becomes more competent. But more importantly, the conscious work of the mathematician "set[s} the unconscious machine in motion" (2008, p. 56). The unconscious procedures are reasonings that go on in the mind of the mathematician even after his focus is turned away from the problem. Among countless possible combinations, only a few can be constructed consciously by the mathematician, which, in most cases, will appear to be barren and useless in the beginning. But Poincaré observes that the unconscious ego, or the "subliminal ego", once stirred by a conscious work, would keep reasoning about the problem by eliminating the useless and encumbering combinations among the extremely numerous possibilities, selecting the most fruitful ones. What guides the mathematician in this unconscious work is an intuition that is certainly not operating arbitrarily, but following extremely subtle and delicate rules, which are practically impossible to be stated in precise language; "They must be felt rather than formulated" (2008, p. 57). What is felt here is the "mathematical beauty; of the harmony of numbers and forms and of geometric elegance. It is a real aesthetic feeling that all true mathematicians recognize" (p. 59). Poincaré states that the results of this unconscious work present themselves to the mind in moments of sudden illumination.

What we need to notice is that the intuition Poincaré described in these passages is not exclusive to the mathematician. In fact, there are other philosophers who have noted that this intuition is not limited to mathematics. Folina (1986) regards it as a faculty that glosses over the incomplete character of both mathematical and empirical experience (p. 86). Gerhard Heinzmann (1998) likens it to "the awareness of the mastery of a schema" (p. 48). We can easily assume the mathematician in the above paragraph being replaced with a composer and the proof with a musical piece that the composer is working on. Similarly, we can imagine a chess player who is trying to devise a new and unheard of strategy. Analogous to Poincaré's mathematician, the composer would most likely say that what guides her in the process of creation is a certain kind of intuition. We can imagine her saying that through this intuition, which is very difficult to elucidate, she appreciates harmony and is able to recognize the patterns that arouse in her a feeling of beauty, and what she is looking for very often comes to her in moments of inspiration. This kind of intuition is also what distinguishes a professor from a student of mathematics. Surely, what causes the difference between them is primarily experience, but the abundance of experiences would be useless if it did not help to develop in the mathematician a certain kind of intuition, i.e. the feeling towards mathematical beauty. Because this feeling is highly improved and polished in a professor, he would not waver in the face of a problem that would easily overwhelm a student.

If intuition in mathematics was confined solely to what is described here then it would truly be something entirely psychological, not something over and above the intuition developed in a specialist concerning his or her particular field. However, even though the student of mathematics – or an amateur in any particular field – lacks this kind of intuition compared to a specialist, both the student and the professor of mathematics share something in common: *both possess a mind that is capable of conceiving the idea of indefinite repetition*, and therefore able to count. They both have a direct intuition of this capacity, and as Poincaré wrote, experience is only an opportunity for them of using it. Because the student also has this capacity, he is able to construct numbers just like the professor, and the reasoning behind the method of proof by mathematical induction is not going to be a mystery for him. Through countless experiences the student would eventually develop the kind of intuition described in the previous paragraphs, but he would never have learned mathematics and understood mathematical reasoning if he did not have an intuition of this distinctive mental capacity in the first place, which amounts basically to the ability to count indefinitely; without it, the concept of 'number' would be completely meaningless and empty.

There are then two kinds of intuition. First, there is the intuition that helps a specialist, which we will call *sensible intuition*; second, there is the *pure* intuition that gives rise to mathematical thinking, which is basically the intuition of a mental capacity. Poincaré offered a reliable criterion for distinguishing these two kinds of intuition. He wrote that there are intuitions that may deceive us, and then there are intuitions that may never do so. For instance, the intuition a chess player has developed may sometimes deceive him. We can imagine the game he planned being outmaneuvered by a more brilliant strategy coming from his opponent. The mathematician guided by feelings towards mathematical beauty also "need[s] to work out the results of the inspiration" (2008, p. 56). A more striking example that Poincaré gives of deceptive intuition is *geometric intuition*. He writes: "If we try to imagine a line [...] our representation must have a certain breadth. Two lines will therefore appear to us under the form of two narrow bands". The geometer "[imagines] a line as the limit towards which tends a band that is growing thinner and thinner, and the point as the limit towards which is tending an area that is growing smaller and smaller" (2011, p. 31). On the basis of our representations we intuitively conclude that whenever we imagine a curve, there are going to be an infinite number of lines intersecting this curve at only one point, i.e. that the function this curve represents will be everywhere differentiable. By establishing the existence of functions which are continuous everywhere but differentiable nowhere, Karl Weierstrass showed once again that we were mistaken in trusting our intuitions in geometry. What the chess player and the geometer have in common is the fact that the basis of their intuition is *experience*: the player's intuition depends upon the countless games he has played, and the geometer's intuition depends upon the countless observations she has made of the motion of the most notable objects around her, in our case, solid bodies. The role that experience plays in geometry and how the motion of solid bodies is related to it are subjects that are too extensive to be covered in this paper[6]. At the moment it is sufficient to know that Poincaré argued that geometry is not completely independent of experience and this is why geometric intuition sometimes deceives us.

On the other hand, the intuition we have of our capacity to iterate indefinitely – and consequently, of the principle of mathematical induction – can never deceive us, because "it is only the affirmation of a property of the mind itself" (2011, p. 17). Poincaré called this "the intuition of pure number" (1907, p. 20). Nothing empirical plays a role in formulating the principle of mathematical induction; its truth is known *a priori* on the basis of a pure intuition and it is "imposed upon us with such an irresistible weight of evidence" (2011, p. 16), for what is intuited here is simply a mental capacity. The term intuition is used both for that which makes us conscious of a distinctive mental capacity and also that which makes us conscious of the strategy behind any practice, giving us competence – be it in chess, mathematics, or even logic. This is because in both cases we become conscious of the object of our inquiry *immediately,* without surveying all the intermediary steps. Our intuition is "an incomplete summary of a piece of true reasoning" (2011, p. 216), or in Folina's words, a glossing-over faculty. The difference is that in pure intuition what is summarized is an *a priori* reasoning, whereas in the intuition that a practitioner (or a geometer) develops what is summarized is a reasoning that is derived from experience. This is precisely why we can never be deceived by pure intuition but we may be misled by sensible intuition. However, when we use the term sensible intuition in the context of Poincaré's philosophy, it should be noted that this is not quite in line with Kant's conception. Both philosophers took this to mean an intuition that is derived from experience (though in Kant's case, this already presupposes a pure, *a priori* form of sensibility) and, as such, incapable of being a basis for necessary

[6] For more information about Poincaré's views on the relationship between geometry and experience and on the significance of solid body motion, see Poincaré (2011) pp. 60-100; (1907) pp. 37-74.

and universal truths. Unlike Kant, Poincaré thought that the intuition that lies at the basis of geometry is not pure; it is to a certain degree sensible. He believed that the intuition we have of our forms of sensibility is not pure, because our forms of sensibility – the mental frameworks to which objects conform – are not given *a priori*; these are convenient frameworks invented by us under the guidance of experience[7]. This is perhaps the most fundamental difference that separates Poincaré from Kant and it is the origin of Poincaré's stronger empiricist tendencies.

One final remark on the difference between the intuition developed in a specialist and the intuition of pure number: it may be argued that this distinction is unnecessary, for these two intuitions can be reduced to one. There is no doubt that these intuitions are related to a certain degree. A very important element of the intuition that helps a professional – be it a chess player or a mathematician – in his or her particular field, is the recognition of patterns that are repeating. Recognition of these repeating patterns is actually constitutive of the "feeling of beauty" that guides a specialist. Thus, it would not be wrong to say that the idea of repetition is an integral part of the intuition developed in a specialist. How these two intuitions are related and whether they can be reduced to one is beyond the scope of this paper. However, it is worth noting that we cannot fully explain the intuition in mathematics in terms of the intuition developed in a specialist; as we have said, the latter is sometimes deceptive. Contrary to this, according to Poincaré, the intuition of pure number on

[7] Poincaré does not claim that the idea of space is completely derived from experience. There are certainly *a priori* elements in geometry, and it is these elements which can account for the necessity in geometric truths. One of these elements is the ability to conceive of the idea of indefinite repetition, which is discussed in detail in this paper. Another perhaps more fundamental *a priori* element in geometry is the idea of a mathematical *group* (in modern terms, a set with a binary operation that satisfies group axioms). Poincaré argues that space is actually *a group of displacements* and there are different possible groups for representing the motion of an invariable figure. There are different geometries each corresponding to a particular group. When space is assumed to have three dimensions, there are a limited number of possible geometries: constant negative curvature, constant positive curvature, and zero curvature (i.e. the Euclidean group). For Poincaré, the role of experience is to help us chose among these groups the "one that will be the standard, so to speak, to which we shall refer natural phenomena" (2011, p.82). Neither geometry (and neither group) is the *true* geometry; one geometry can only be more convenient than another. For a detailed discussion of groups and how experience guides us in adopting the most convenient one among other possible options see Poincaré (1898), pp. 7-34; (2011), pp. 42-83.

which arithmetic is grounded can never deceive us. What this intuition amounts to is the capacity to conceive of the idea of indefinite repetition. From repetition, number arises, and the principle of mathematical induction is simply the affirmation of this mental capacity.

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Öz: Bu makalenin hedefi Henri Poincaré'nin sezgiciliğine ışık tutmaktır. Bugün matematik felsefesinde sezgicilik görüşü, çoğunlukla L.E.J. Brouwer ile ve üçüncü hâlin imkansızlığı ilkesine dayanan kanıtları matematikten çıkarma fikri ile ilişkilendirilmektedir. Sezgicilerin, matematiksel yöntemimizde bir kusur

bulunduğu ve köklü bir değişim gerektiği iddiasında olduklarına dair yaygın bir kanı vardır. Fakat şaşırtıcıdır ki matematiği insanoğlunun sezgilerine bağlayan ilk düşünür Immanuel Kant'ın böyle bir iddiası olmamıştır ve kendisi sezgi terimini bugün anlaşıldığından farklı bir anlamda kullanmıştır. Bu, Kant'ın felsefesinde kayda değer değişiklikler yapan ve bir ön-sezgici veya yarı-sezgici olarak anılan Poincaré için de geçerlidir. Warren Goldfarb gibi bazı filozoflar haklı olarak Poincaré'nin sezgiye başvurmasının sebebinin matematiksel düşünmenin psikolojik boyutunu anlatmak olduğunu iddia etmişlerdir. Bu makalede, bahsi geçen psikolojik boyutun Poincaré'nin sezgiciliğinin tümü olmadığı ve felsefesinde Kantçı gelenekten aldığı a priori, saf bir sezgi fikri olduğu savunulmaktadır.

Anahtar Kelimeler: Sezgicilik, Kant, Poincaré, sentetik a priori, matematiksel tümevarım.